Bending of Beams

e-content for B.Sc Physics (Honours)
B.Sc Part-I

Paper-I

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Bending of Beams

Beams: A beam is defined as a rod or bar. Circular or rectangular of uniform cross section whose length is very much greater than its other dimensions, such as breadth and thickness. It is commonly used in the construction of bridges to support roofs of the buildings etc. Since the length of the beam is much greater than its other dimensions the shearing stresses are very small.

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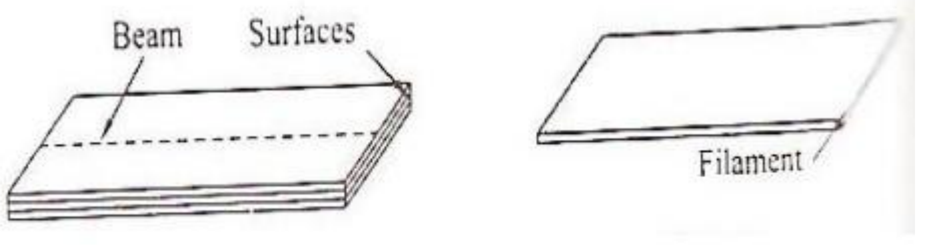
Assumptions:

While studying about the bending of beams, the following assumptions have to be made.

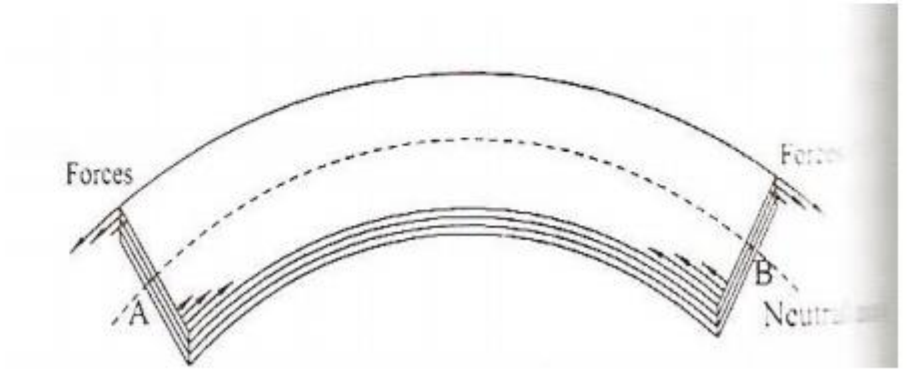
1. The length of the beam should be large compared to other dimensions.
2. The load(forces) applied should be large compared to the weight of the beam
3. The cross section of the beam remains constant and hence the geometrical moment of inertia i_g also remains constant
4. The shearing stresses are negligible
5. The curvature of the beam is very small

Bending of a Beam and neutral axis

Let us consider a beam of uniform rectangular cross section in the figure. A beam may be assumed to consist of a number of parallel longitudinal metallic fibers placed one over the other and are called as filaments as shown in the figure.

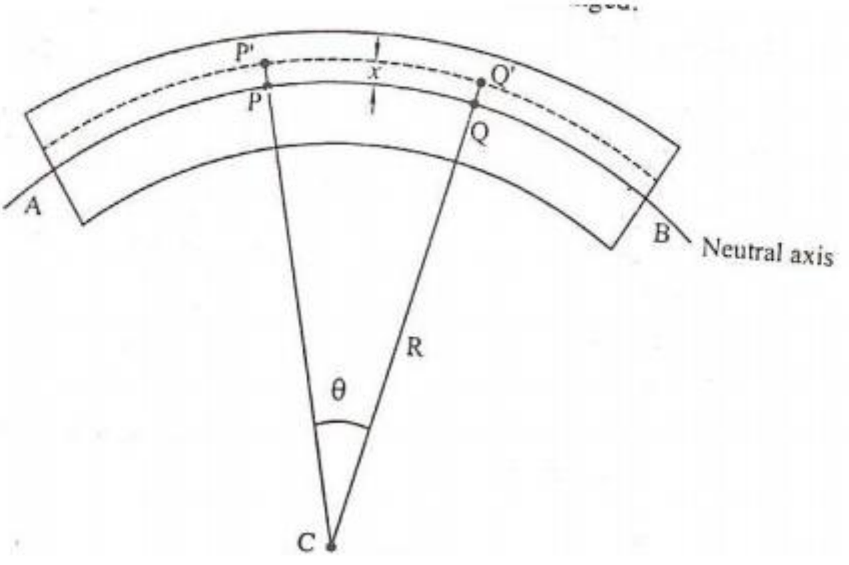


Let the beam be subjected to deforming forces as its end as shown in the figure. Due to the deforming force the beam bends. We know the beam consist of many filaments. Let us consider a filament AB at the beam. It is found that the filaments(layers) lying above AB gets elongated, while the filaments lying below AB gets compressed. Therefore the filaments i.e layer AB which remains unaltered is taken ass the reference axis called neutral axis and the plane is called neutral plane. Further, the deformation of any filaments can be measured with reference to the neutral axis.



EXPRESSION FOR BENDING MOMENT

Let us consider a beam under the action of deforming forces. The beam bends into a circular arc as shown in the figure. Let AB be the neutral axis of the beam. Here the filaments above AB are elongated and the filaments below AB are compressed. The filament AB remains unchanged.



Let PQ be the chosen from the neutral axis. If R is the radius of curvature of the neutral axis and θ is the angle subtended by it at its center of curvature'C'

Then we can write original length

$PQ=R^{\theta}$ ………………………………………………………… 1
Let us consider a filament P'Q' at a distance 'X' from the neutral axis.
We can write extended length
$P'Q'=(R+x)^{\theta}$ ………………………………………………2
From equations 1 and 2 we have,
Increase in length=P'Q'-PQ
On increase in its length=(R=x)θ-Rθ
Increase in length=xθ ……………………………………3
We know linear strain=increase in length\original length
Linear strain=xθ\Rθ=x\R ………………………………4
We know, the youngs modulus of the material
Y=stress\linear strain
Or
stress=y*linear strain ……………………5
Substituting 4 in 5, we have
Stress=Yx\R
If δA is the area of cross section of the filament P'Q', then,
The tensile force on the area δA=stress*Area
Ie. Tensile force=(Yx\R).δa
We know the memont of force= force*Perpendicular distance
Moment of the tensile force about the neutral axis AB
or

$$PQ = \frac{Yx}{R}.\delta A.x$$

$$PQ = \frac{Y}{R}.\delta A.x^2$$

The moment of force acting on both the upper and lower halves of the neutral axis can be got by summing all the moments of tensile and compressive forces about the neutral axis

$$\therefore \text{ The moment of all the forces about the neutral axis} = \frac{Y}{R}..\sum x^2 \delta A$$

Here $I_g = \sum x^2 \delta A = AK^2$ is called as the geometrical moment of inertia.

Where, A is the total area of the beam and K is the radius of Gyration.

$$\therefore \text{Total Moment of all the forces Or Internal bending Moment} = \frac{YI_g}{R} \longrightarrow 6$$

SPECIAL CASES

a) Rectangular Cross section

If 'b' is the breadth and 'd' is the thickness of the beam, them

Area A = bd and $K = \frac{d^2}{12}$

$$\therefore I_g = AK^2 = \frac{bd.d^2}{12} = \frac{bd^3}{12}$$

Substituting the value of I_g in equation 6, we can write

Bending moment for rectangular across section = = $\frac{bd^3}{12R}$ → 7

b) Circular Cross Section

For a circular cross section if 'r' is the radius, then Area A = πr^2 and $K^2 = \frac{r^2}{4}$

$$I_g = AK^2 = \frac{\pi r^2 \times r^2}{4} = \frac{\pi r^4}{4}$$

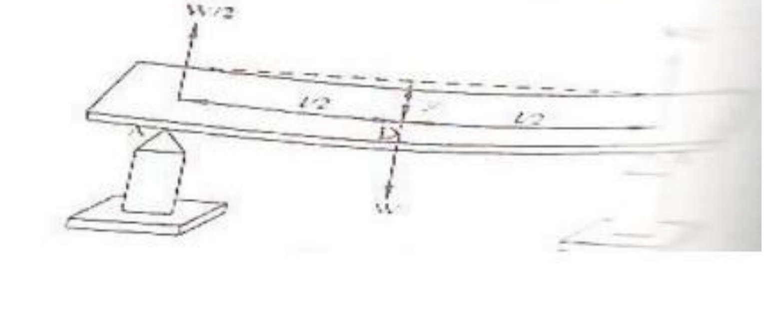
Substituting the value of I_g in equation 6 we can write

The Bending moment of a circular cross section = $\frac{\pi Y r^4}{4R}$ → 8

NON-UNIFORM BENDING-DEPRESSION OF THE MID POINT OF A BEAM LAODED AT THE MIDDLE THEORY

Let us consider a beam of length 'l' (distance between the two knife edges) supported on the two knife edges A and B as shown in the figure. The load of weight 'W' is suspended at the centre 'C'. It is found that the beam bends and the maximum displacement is at the point 'D'Where the load is given.

Due to the load (W) applied, at the middle of the beam the reaction W/2 is acted vertically upwards at each knife edges. The bending is celled Non-Uniform bending



The beam may be considered as two cantilevers, whose free end carries a load of W/2 and fixed at the point 'D'.
Hence we can say the elevation of A above D as the depression below 'A'. We know the depression of a cantilever

$$y = \frac{Wl^3}{3YI_g} \longrightarrow 1$$

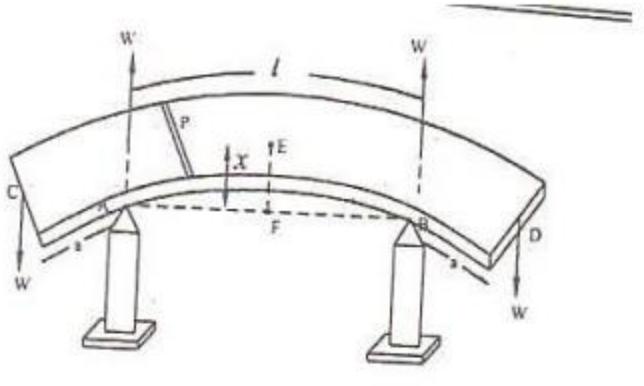
Therefore substituting the value l and l/2 and was W/2 in the expression for the depression of the cantilever we have

$$y = \frac{(W/2)(l/2)^3}{3YI_g} \longrightarrow 2$$

Or $$y = \frac{Wl^3}{48YI_g} \longrightarrow 3$$

UNIFORM BENDING-ELEVATION AT THE CENTER OF THE BEAM LOADED AT BOTH THE ENDS THEORY:

Let us consider a beam of negligible mass, supported symmetrically on the two knife edges A and B as shown. Let the length between A and B is'l'. Let equal weights W; be added to either end of the beam C and D.



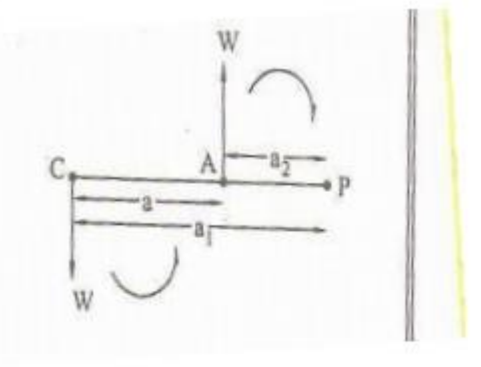
Let CA=BD
Due to load applied the beam bends from position F and e into an arc of a circle and produces as elevation 'x' from position F and E. Let 'W' be the reaction produced at the points A and B acts vertically upwards as shown in figure.
Consider a point 'P' on the cross section of the beam. Then the forces acting on the part PC of the beam are
a) Force W at 'C' and

b) Reaction W at A as shown in the figure

Let he distance PC = a_1 and PA = a_2, then

The external bending moment about 'P' is

$$M_p = W \times a_1 - W \times a_2$$



Here the clockwise moment is taken as negative and anticlockwise moment is taken as positive.

External bending moment about P can be written as

$$M_p = W \times (a_1 - a_2)$$

$$M_p = Wa \longrightarrow \quad 1$$

We know the internal bending moment = $\frac{YI_g}{R} \longrightarrow \quad 2$

Under equilibrium condition

External bending moment = Internal bending moment

External bending moment = Internal bending moment

We can write Equation 1 = Equation 2

$$Wa = \frac{YI_g}{R} \longrightarrow \quad 3$$

Since for a given load (W) Y, Ig and R are constant the bending is called Bending. Here it is found that the elevation 'x' forms an arc of the circle of radius 'R', as shown in the figure.

From the ΔAFO we can write

$$OA^2 = AF^2 + FO^2$$

Since OF = FE, therefore we can write

$$OA^2 = AF^2 + FE^2$$
$$AF^2 = OA^2 - FE^2$$

Rearranging we can write

$$AF^2 = FE\left[\frac{OA^2}{FE} - FE\right] \longrightarrow \quad 4$$

Here AF = l/2 ; FE = x= R/2; OA = R

$\therefore$ Equation 4 can be written as

$$\left(\frac{l}{2}\right)^2 = x\left[\frac{R^2}{(R/2)} - x\right]$$
$$\frac{l^2}{4} = x[2R - x]$$
$$\frac{l^2}{4} = 2xR - x^2$$

If the elevation is very small, then the term x^2 can be neglected.

$\therefore$ We can write $\frac{l^2}{4} = 2xR$

Or $x = \frac{l^2}{8R}$

$\therefore$ Radius of the curvature $R = \frac{l^2}{8x} \longrightarrow \quad 5$

Substituting the value of 'R' value in equation 3, we have

$$W.a = \frac{YI_g}{\left(l^2/8x\right)}$$

$$W.a = \frac{8YI_g x}{l^2} \longrightarrow \qquad 6$$

Rearranging equation 6 we have

The elevation of Point 'E' above 'A' is $x = \frac{Wal^2}{8YI_g}$